L-functions

(PARI-GP version 2.13.1)

Characters

A character on the abelian group $G = \sum_{j \leq k} (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$, e.g. from `znstar(q,1)` $\leftrightarrow (\mathbf{Z}/q\mathbf{Z})^*$ or `bnrinit` $\leftrightarrow \mathrm{Cl}_{\mathfrak{f}}(K)$, is coded by $\chi = [c_1, \ldots, c_k]$ such that $\chi(g_j) = e(c_j/d_j)$. Our L-functions consider the attached *primitive* character.

Dirichlet characters $\chi_q(m, \cdot)$ in Conrey labelling system are alternatively concisely coded by `Mod(m,q)`. Finally, a quadratic character $(D/\cdot)$ can also be coded by the integer D.

L-function Constructors

An `Ldata` is a GP structure describing the functional equation for $L(s) = \sum_{n \geq 1} a_n n^{-s}$.
- Dirichlet coefficients given by closure $a : N \mapsto [a_1, \ldots, a_N]$.
- Dirichlet coefficients $a^*(n)$ for dual L-function L^*.
- Euler factor $A = [a_1, \ldots, a_d]$ for $\gamma_A(s) = \prod_i \Gamma_{\mathbf{R}}(s + a_i)$,
- classical weight k (values at s and $k - s$ are related),
- conductor N, $\Lambda(s) = N^{s/2} \gamma_A(s)$,
- root number ε; $\Lambda(a, k - s) = \varepsilon \Lambda(a^*, s)$.
- polar part: list of $[\beta, P_\beta(x)]$.

An `Linit` is a GP structure containing an `Ldata` L and an evaluation *domain* fixing a maximal order of derivation m and bit accuracy (`realbitprecision`), together with complex ranges
- for L-function: $R = [c, w, h]$ (coding $|\Re z - c| \leq w$, $|\Im z| \leq h$); or $R = [w, h]$ (for $c = k/2$); or $R = [h]$ (for $c = k/2$, $w = 0$).
- for θ-function: $T = [\rho, \alpha]$ (for $|t| \geq \rho$, $|\arg t| \leq \alpha$); or $T = \rho$ (for $\alpha = 0$).

Ldata constructors

Riemann ζ	`lfuncreate(1)`
Dirichlet for quadratic char. $(D/\cdot)$	`lfuncreate(D)`
Dirichlet series $L(\chi_q(m, \cdot), s)$	`lfuncreate(Mod(m,q))`
Dedekind ζ_K, $K = \mathbf{Q}[x]/(T)$	`lfuncreate(bnf)`, `lfuncreate(T)`
Hecke for χ mod $\mathfrak{f}$	`lfuncreate([bnr, χ])`
Artin L-function	`lfunartin(nf, gal, M, n)`
Lattice Θ-function	`lfunqf(Q)`
From eigenform F	`lfunmf(F)`
Quotients of Dedekind η: $\prod_i \eta(m_{i,1} \cdot \tau)^{m_{i,2}}$	`lfunetaquo(M)`
$L(E, s)$, E elliptic curve	`E = ellinit(...)`
$L(Sym^m E, s)$, E elliptic curve	`lfunsympow(E, m)`
genus 2 curve, $y^2 + xQ = P$	`lfungenus2([P, Q])`
dual L function $\hat{L}$	`lfundual(L)`
$L_1 \cdot L_2$	`lfunmul(L_1, L_2)`
L_1/L_2	`lfundiv(L_1, L_2)`
$L(s - d)$	`lfunshift(L, d)`
$L(s) \cdot L(s - d)$	`lfunshift(L, d, 1)`
twist by Dirichlet character	`lfuntwist(L, χ)`
low-level constructor	`lfuncreate([a, a^*, A, k, N, eps, poles])`
check functional equation (at t)	`lfuncheckfeq(L, {t})`

Linit constructors

initialize for L	`lfuninit(L, R, {m = 0})`
initialize for θ	`lfunthetainit(L, {T = 1}, {m = 0})`
cost of `lfuninit`	`lfuncost(L, R, {m = 0})`
cost of `lfunthetainit`	`lfunthetacost(L, T, {m = 0})`
Dedekind ζ_L, L abelian over a subfield	`lfunabelianrelinit`

L-functions

L is either an `Ldata` or an `Linit` (more efficient for many values).

Evaluate

$L^{(k)}(s)$	`lfun(L, s, {k = 0})`
$\Lambda^{(k)}(s)$	`lfunlambda(L, s, {k = 0})`
$\theta^{(k)}(t)$	`lfuntheta(L, t, {k = 0})`
generalized Hardy Z-function at t	`lfunhardy(L, t)`

Zeros

order of zero at $s = k/2$	`lfunorderzero(L, {m = -1})`
zeros $s = k/2 + it$, $0 \leq t \leq T$	`lfunzeros(L, T, {h})`

Dirichlet series and functional equation

$[a_n : 1 \leq n \leq N]$	`lfunan(L, N)`
conductor N of L	`lfunconductor(L)`
root number and residues	`lfunrootres(L)`

G-functions

Attached to inverse Mellin transform for $\gamma_A(s)$, $A = [a_1, \ldots, a_d]$.

initialize for G attached to A	`gammamellininvinit(A)`
$G^{(k)}(t)$	`gammamellininv(G, t, {k = 0})`
asymp. expansion of $G^{(k)}(t)$	`gammamellininvasymp(A, n, {k = 0})`